Rough lecture notes for sunrise/sunset and times - Math 550 - Whittlesey

Key definitions:

Celestial equator. The great circle which is the projection of the earth's equator on the celestial sphere.

Ecliptic. The great circle the sun appears to travel through the sky as the earth orbits the sun.

Equinoxes. The intersection points of the ecliptic and celestial equator.

Local meridian. This is the projection of the observer's earth meridian onto the celestial sphere. The local meridian is the meridian where the sun is at its highest altitude during the day.

Apparent solar noon. This is the moment when the sun crosses the local meridian (and reaches its highest altitude during the day.) I will also use this loosely to speak of the moment when the dynamical mean sun (see below) crosses the local meridian.

Apparent solar day. This is the interval between two successive times when the sun crosses the local meridian (apparent solar noon.)

Unfortunate fact: the length of the apparent solar day is not constant; it can vary by as much as 15 minutes. The reasons for this are twofold: (1) The elliptic orbit of the earth means that the sun does not move at a constant speed on the ecliptic and (2) The sun does not cross meridians at a constant rate (i.e., its right ascension does not change at a constant rate) due to the fact that its declination is changing to regions where meridians are closer or further apart, and cuts across meridians at changing oblique angles.)

Mean solar day. Average length of a solar day over the period of one earth orbit around the sun. An hour is one twenty-fourth of a mean solar day.

Dynamical mean sun. This is the position of an object which travels the ecliptic at a constant rate and coincides with the real sun at aphelion and perihelion. In order to simplify things, we're going to pretend that it coincides with the real sun at the equinoxes (this is probably one reason why our sunrise/set calculations don't come out exactly right.)

Fictitious mean sun. This is the position of an object which travels around the celestial equator at the same constant rate as the dynamical mean sun does around the ecliptic. It coincides with the dynamical mean sun at the equinoxes.

Key fact: given two consecutive moments when the fictitious mean sun crosses the local meridian, the time between them is the mean solar day.

Mean solar noon. This is the moment the fictitious mean sun crosses the local meridian (and attains its highest altitude of the day.)

Mean solar time. This is the time measured by setting your clock to 12:00 at mean solar noon.

Greenwich mean time (GMT), or *universal time (UT).* Mean solar time at the meridian of Greenwich, UK.

Pacific Standard Time. Mean solar time of the 120^{th} meridian west of Greenwich.

Local mean time. The mean solar time of your location, e.g., San Marcos.

Procedure for finding sunrise and sunset of dynamical mean sun, pretending it coincides with the real sun at the equinoxes:

1. Find the declination of the sun - which we assume to be constant on a given day. Count the number of days from the latest equinox, and assuming the (dynamical mean) sun travels 59.1 minutes of arc on the ecliptic every day,

figure out how many degrees λ the dynamical mean sun has travelled on the ecliptic from the equinox. Dropping a perpendicular to the celestial equator, use right angle trigonometry to figure the sun's declination (using $\sin(23.5) = \sin(\textit{opposite})/\sin(\textit{hypotenuse})$). You'll also need its right ascension α later, which is the base of the right triangle (and recall $\cos(23.5) = \tan(\textit{adjacent})/\tan(\textit{hypotenuse})$.

2. Use the formula

$$\cos(t) = -\tan(\phi)\tan(\delta) - \frac{\sin(h)}{\cos(\phi)\cos(\delta)},$$

where t is the hour angle at sunrise or sunset; ϕ is your latitude, δ the declination of the sun, and h is 50 minutes of arc. This defines sunrise/sunset to be the moment when the center of the sun is 50 minutes below the horizon; or the top of the sun is 35 minutes below the horizon, so gets refracted to the horizon itself. When you get t in degrees, this gives you the hour angle at sunrise. Dividing by 15 gives you the amount of time in hours between when the dynamical mean sun rises and reaches the local meridian (noon.) Then you can get rise and set for the dynamical mean sun in apparent local time.

3. But we want the rise and set in mean local time. Calculate the difference between the right ascension of the dynamical mean sun and the fictitious mean sun: the fictitious mean sun's RA is λ, but the dynamical mean sun's RA is α (see above). The difference should only be a few degrees. Assuming the earth rotates a degree every 4 minutes, multiply the difference by 4 to figure out how many minutes to adjust the rise and set times to mean local time. You'll have to think carefully about whether to add or subtract the minutes. It turns out that between Dec. 22 and Mar. 22, and again from June 22 to Sept. 22, you have to add minutes (as the dynamical mean sun is ahead of the fictitious mean sun). You subtract minutes from Mar. 22 to June 22 and again from Sept. 22 to Dec. 22.

4. Adjust to Pacific Standard Time: we are 3 degrees ahead of mean local time at the 120^{th} meridian, so subtract 3 times 4, or 12 minutes from the rise and set times, because sunrise and set occur earlier here. If you were in San Francisco, which is 2 degrees west of the 120^{th} meridian, you'd have to add 8 minutes.

Note: derivation of the formula in 2. Done in class, or you can refer to the handout p. 335, where the author does it, except that he ought to replace δ with $-\delta$.