Sec 37

(11) (a) $\dfrac{1}{\sqrt{1-\frac{x^2}{4}}}\cdot\dfrac{d}{dx}\left(\dfrac{x}{2}\right)=\dfrac{1}{\sqrt{1-\frac{x^2}{4}}}\cdot\dfrac{1}{2}=\dfrac{1}{\sqrt{4-x^2}}$ (book is wrong)

(b) $\dfrac{d}{dx}\left(\arccos\dfrac{1-x}{\sqrt{2}}\right)=\dfrac{-1}{\sqrt{1-\left(\frac{1-x}{\sqrt{2}}\right)^2}}\left(-\dfrac{1}{\sqrt{2}}\right)=\dfrac{1}{\sqrt{2-(1-x)^2}}$

(c) $\dfrac{1}{1+\frac{x^4}{a^2}}\left(\dfrac{2x}{a}\right)= = \dfrac{2ax}{a^2+x^4}$

(d) $\dfrac{1}{2}x^{-1/2}-\dfrac{1}{1+x}\left(\dfrac{1}{2}x^{-1/2}\right)=\dfrac{1}{2\sqrt{x}}\left(1-\dfrac{1}{1+x}\right)=\dfrac{1}{2\sqrt{x}}\left(\dfrac{x}{1+x}\right)=\dfrac{\sqrt{x}}{2(1+x)}$

(12) (a) $\dfrac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\cdot\dfrac{d}{dx}\sqrt{1-x^2}=\dfrac{1}{\sqrt{x^2}}\;\dfrac{1}{2}(1-x^2)^{-1/2}(-2x)$

$=\dfrac{-x}{|x|}\,\dfrac{1}{\sqrt{1-x^2}}$

Sec 38

(2) We need $x^2-4>0$, so $x^2>4$, so $|x|>2$.

Domain $=(-\infty,-2)\cup(2,\infty)$

Sec 39

(10) (c) $e^{\sqrt{x+1}}\dfrac{d}{dx}\sqrt{x+1}=e^{\sqrt{x+1}}\dfrac{1}{2}(x+1)^{-1/2}$

(d) $(e^x)'\cos x+e^x(\cos x)'=e^x\cos x-e^x\sin x$

(i) $\dfrac{(x^2+1)(e^x)'-e^x(x^2+1)'}{(x^2+1)^2}=\dfrac{(x^2+1)e^x-2xe^x}{(x^2+1)^2}$

(12) (d) $\frac{1}{x^2-4x}\cdot\frac{d}{dx}(x^2-4x)=\frac{2x-4}{x^2-4x}$

(h) $\frac{(1+\ln x)(1-\ln x)'-(1-\ln x)(1+\ln x)'}{(1+\ln x)^2}=\frac{(1+\ln x)(-\frac{1}{x})-(1-\ln x)(\frac{1}{x})}{(1+\ln x)^2}$

$=\frac{-\frac{1}{x}(1+\ln x+1-\ln x)}{(1+\ln x)^2}=\frac{-\frac{2}{x}}{(1+\ln x)^2}=\frac{-2}{x(1+\ln x)^2}$

(14b) $(x^{\sin x})'=(e^{\sin x\ln x})'=e^{\sin x\ln x}\cdot(\sin x\ln x)'$

$=x^{\sin x}\cdot\left(\cos x\ln x+(\sin x)\cdot\frac{1}{x}\right)$

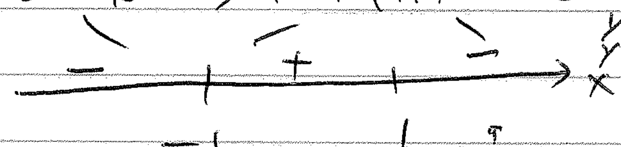
HW #8 solutions (evens)

Sec 41 ② The function f isn't diff'ble at $x=0$, so Rolle does not apply.

⑩ Suppose $x+1 > x > 25$.
Then $\sqrt{x+1} - \sqrt{x} = \frac{1}{2\sqrt{c}}\left((x+1) - x\right)$ by the MVT,

where c is between x and $x+1$ (so $c > 25$.
This, in turn $= \frac{1}{2\sqrt{c}} < \frac{1}{2\sqrt{25}}$ since $c > 25$
$= \frac{1}{10}$, as desired.

⑱ⓑ $y' = 3 - 3x^2 = 3(1-x)(1+x) = 0$ when $x = \pm 1$

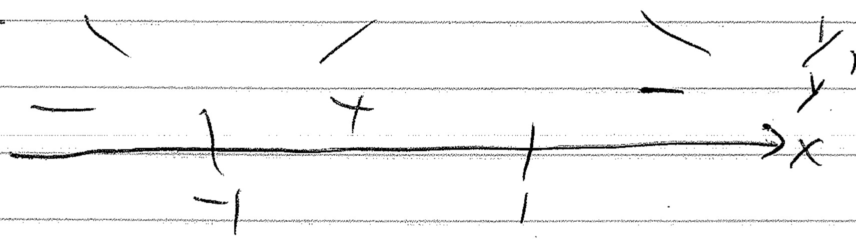


$y'(-2) < 0$ $\quad y'(0) = 3 > 0$ $\quad y'(2) < 0$

y decreases on $(-\infty, -1)$, $(1, \infty)$
increases on $(-1, 1)$.

⑱ⓒ $y' = \dfrac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2} = \dfrac{2(1+x^2-2x^2)}{(1+x^2)^2}$

$= \dfrac{2(1-x)(1+x)}{(1+x^2)^2}$



Decreases on $(-\infty, -1)$, $(1, \infty)$

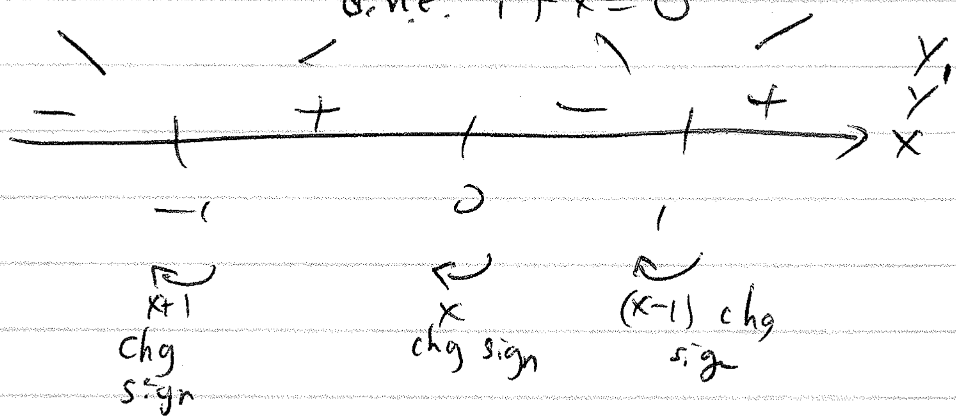
Increases on $(-1, 1)$

Domain $: \mathbb{R} \setminus \{0\}$

(19d) $y' = 2x - \frac{2x}{x^2} = \frac{2x(x^2-1)}{x^2} = \frac{2x(x+1)(x-1)}{x^2}$

$= \frac{2(x+1)(x-1)}{x} = 0$ if $x = \pm 1$

d.n.e. if $x = 0$



Increasing: $(-1,0)$, $(1,\infty)$ Decreasing: $(-\infty,-1)$, $(0,1)$.

(26) The proof of Thm 6.19 holds as is, as long as we can show that at a place c in (a,b) where f attains a maximum or minimum, we cannot have $f'(c) = \pm\infty$.

Suppose f has a max when $x=c$ and $f'(c) = +\infty$. Then $\lim_{x\to c^+} \frac{f(x)-f(c)}{x-c} = +\infty$

$\Rightarrow \frac{f(x)-f(c)}{x-c} > 0$ for x in some interval $(c, c+\delta)$.

But for these x, $x-c>0$, so $\left(\frac{f(x)-f(c)}{x-c}\right)(x-c) > 0$,

So $f(x)-f(c) > 0 \Rightarrow f(x) > f(c)$, contradicting the assumption that f has a (local) max at $x=c$.

So we cannot have $f'(c) = +\infty \Rightarrow f'(c)$ exists.

The case $f'(c) = -\infty$ is similar (look at $\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c}$)

(The cases where f has a min at c are also similar.)