Math 430 HW

Prove $\lim_{x\to 0} \frac{4}{\sin x} = \infty$

Know	Want

$0 < |x| < \delta$

$\delta = 4/M$

$|\sin x| \le |x| \longrightarrow |\sin x| < 4/M$

need $\sin x \neq 0$ so δ must $< \pi$

$\left|\frac{4}{\sin x}\right| > M$

Pf

Fix $M > 0$. Let $\delta = \min\{\pi, 4/M\}$. Then if $0 < |x| < \delta$,

$0 < |x| < \pi \Rightarrow \sin x \neq 0 \Rightarrow |\sin x| > 0$.

Also $0 < |x| < 4/M \Rightarrow$ (since $|\sin x| \le x$) $0 < |\sin x| < 4/M$.

So $\frac{1}{|\sin x|} > \frac{M}{4} \Rightarrow \left|\frac{4}{\sin x}\right| > M$.

Since M was arbitrary, $\lim_{x\to 0} \frac{4}{\sin x} = \infty$ by definition of limit.

Prop If $\lim_{x \to a} f(x) = +\infty$, $\lim_{x \to a} g(x) = c > 0$

then $\lim_{x \to a} f(x) g(x) = +\infty$

Pf

Know	Want
$0 < \lvert x-a \rvert < \delta_1 \Rightarrow f(x) > M$	
$0 < \lvert x-a \rvert < \delta_2 \Rightarrow$ $g(x) > c/2$ $\longrightarrow$	$f(x)g(x) > M\frac{c}{2}$
	$f(x)g(x) > M$

need to go back to chg M to $\frac{2M}{c}$

Pf

Fix $M > 0$. Since $\lim_{x \to a} f(x) = +\infty$ $\exists$ $\delta_1 > 0 \ni$

if $0 < |x-a| < \delta_1$ then $f(x) > \frac{2M}{c}$. Since $\lim_{x \to a} g(x) = c > 0$,
$\exists$ $\delta_2 > 0 \ni$ $0 < |x-a| < \delta_2 \Rightarrow g(x) > c/2$.

Let $\delta = \min\{\delta_1, \delta_2\}$. Then if $0 < |x-a| < \delta$,
$0 < |x-a| < \delta_1$ and $0 < |x-a| < \delta_2$, so
$f(x) > 2M/c > 0$ and $g(x) > c/2 > 0$.
So $f(x) g(x) > \frac{2M}{c} \cdot c/2 = M$
Since M was arbitrary, $\lim_{x \to a} f(x) g(x) = +\infty$ by definition.